Solution To Quiz 1 DIS 210

Problem 1:

$$(\underline{a_1}, \underline{a_2}, \underline{a_3}, \underline{b}) = \begin{pmatrix} 1 & 3 & 4 & 11 \\ 2 & 2 & 9 & 15 \\ 3 & 1 & 7 & 12 \end{pmatrix} \xrightarrow[③-①\times3]{②-①\times2} \begin{pmatrix} 1 & 3 & 4 & 11 \\ 0 & -4 & 1 & -7 \\ 0 & -8 & -5 & -21 \end{pmatrix}$$

$$\xrightarrow{③-②\times2} \begin{pmatrix} 1 & 3 & 4 & 11 \\ 0 & -4 & 1 & -7 \\ 0 & 0 & -7 & -7 \end{pmatrix} \xrightarrow{③/(-7)} \begin{pmatrix} 1 & 3 & 4 & 11 \\ 0 & -4 & 1 & -7 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow[\substack{①-③\times4 \\ ②-③}]{\text{Go backward}} \begin{pmatrix} 1 & 3 & 0 & 7 \\ 0 & -4 & 0 & -8 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{②/(-4)} \begin{pmatrix} 1 & 3 & 0 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{①-②\times3} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

So we get $\underline{b} = \underline{a_1} + 2\underline{a_2} + \underline{a_3}$

Problem 2:

$$\begin{pmatrix} 1 & 2 & a & 1 \\ -1 & 0 & 1 & 1 \\ a & -1 & 1 & 1 \end{pmatrix} \xrightarrow{\text{flip } ①②} \begin{pmatrix} -1 & 0 & 1 & 1 \\ 1 & 2 & a & 1 \\ a & -1 & 1 & 1 \end{pmatrix} \xrightarrow[③+a\times①]{①+②} \begin{pmatrix} -1 & 0 & 1 & 1 \\ 0 & 2 & a+1 & 2 \\ 0 & -1 & a+1 & a+1 \end{pmatrix}$$

$$\xrightarrow{②/2} \begin{pmatrix} -1 & 0 & 1 & 1 \\ 0 & 1 & \frac{a+1}{2} & 1 \\ 0 & -1 & a+1 & a+1 \end{pmatrix} \xrightarrow{③+②} \begin{pmatrix} -1 & 0 & 1 & 1 \\ 0 & 1 & \frac{a+1}{2} & 1 \\ 0 & 0 & \frac{3(a+1)}{2} & a+2 \end{pmatrix}$$

So if $\frac{3(a+1)}{2} \neq 0$ that means $a \neq -1$ then $\frac{3(a+1)}{2} x_3 = a+2$

$$\Rightarrow x_3 = \frac{2(a+2)}{3(a+1)}$$

go backward solve for x_2, x_1 to get unique solution

if $a+1=0$

we get $a=-1$ so $0 x_3 = 1$

Contradiction No Solution!