Calculus 1B Quiz 10 Name:

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1. (1 point) Evaluate

$$\int_0^{\frac{\pi}{2}} \sin^3(4x)dx$$

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin^3(4x)dx &= \int_0^{\frac{\pi}{2}} (1-\cos^2(4x))\sin(4x)dx \\ &= -\frac{1}{4}\int_1^1 (1-u^2)du, \quad \text{where } u = \cos(4x) \\ &= 0\end{aligned}$$

since the bounds of integration are equal. [remember to change the bounds! It can save you time and work].

2. (1 point) Integrate

$$\int \sec^9(x)\tan^3(x)dx$$

There are an odd number of $\tan(x)$, so we want to set $u = \sec(x)$.

$$\begin{aligned}\int \sec^9(x)\tan^3(x)dx &= \int u^8\tan^2(x)du \\ &= \int u^8(\sec^2(x)-1)du \\ &= \int u^{10} - u^8 du \\ &= \frac{u^{11}}{11} - \frac{u^9}{9} + C \\ &= \frac{\sec^{11}(x)}{11} - \frac{\sec^9(x)}{9} + C\end{aligned}$$

(turn over)

3. (1 point) Integrate

$$2\int \sin(2x)\cos(2x)e^{2\cos^2(x)}dx$$

We need to get the cos and sin to either all have x or all have $2x$. So we use the double-angle formula $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$ to get everything in terms of $2x$. So:

$$\begin{aligned}
2\int \sin(2x)\cos(2x)e^{2\cos^2(x)}dx &= 2\int \sin(2x)\cos(2x)e^{2(1+\cos(2x))/2}dx \\
&= 2e\int \sin(2x)\cos(2x)e^{\cos(2x)}dx \\
&= -e\int ue^{u}du, \quad \text{where } u = \cos(2x) \\
&= -e\left[ue^{u} - \int e^{u}du\right], \quad \text{integration by parts with } f = u \text{ and } g' = e^{u}du \\
&= e^{1+u} - ue^{1+u} + C \\
&= e^{1+\cos(2x)} - \cos(2x)e^{1+\cos(2x)} + C
\end{aligned}$$