Calculus 1B Quiz 10 Name:

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1. (1 point) Evaluate

$$\int_0^{\frac{\pi}{3}} \sin^2(3x)dx$$

$$\begin{aligned}\int_0^{\frac{\pi}{3}} \sin^2(3x)dx &= \frac{1}{2}\int_0^{\frac{\pi}{3}} (1-\cos(6x))dx \\ &= \left[\frac{x}{2} - \frac{\sin(6x)}{12}\right]\Bigg|_0^{\frac{\pi}{3}} \\ &= \frac{\pi}{6}\end{aligned}$$

2. (1 point) Integrate

$$\int \sec^4(x)\tan^{11}(x)dx$$

Even power of sec, so we'll save one $\sec^2$ and convert the rest into tan. (We also could have noticed that it's an odd power of tan and saved a $\sec\tan$ and converted all the remaining tan into sec, but that would yield a very large polynomial that would be annoying to expand).

$$\begin{aligned}\int \sec^4(x)\tan^{11}(x)dx &= \int \tan^{11}(x)(\tan^2(x)+1)\sec^2(x)dx \\ &= \int u^{11}(u^2+1)du, \quad \text{where } u=\tan(x) \\ &= \frac{u^{14}}{14} + \frac{u^{12}}{12} + C \\ &= \frac{\tan^{14}(x)}{14} + \frac{\tan^{12}(x)}{12} + C\end{aligned}$$

(turn over)

3. (1 point) Integrate

$$\int \frac{(1+\cos(2x))^2}{\cos(x)} dx$$

The double angle formula tells us that $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$, so we will use this to simplify the expression.

$$\begin{aligned}
\int \frac{(1+\cos(2x))^2}{\cos(x)} dx &= \int \frac{(2\cos^2(x))^2}{\cos(x)} dx \\
&= 4\int \cos^3(x) dx \\
&= 4\int (1-\sin^2(x))\cos(x) dx \\
&= 4\int (1-u^2) du, \quad \text{where } u = \sin(x) \\
&= 4u - \frac{4}{3}u^3 + C \\
&= 4\sin(x) - \frac{4}{3}\sin^3(x) + C
\end{aligned}$$