Calculus 1B Quiz 7 Name:

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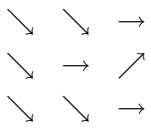
1. (1 point) Draw a direction field for the differential equation $y' = x - y^2$, only on the square consisting of the 9 points: $(-1,-1), (-1,0), (-1,1), (0,-1), (0,0), (0,1), (1,-1), (1,0), (1,1)$.

The direction field (without lines for the axes) would like like the one below, except the two lines in the left-hand corners would have even steeper downward slopes.



2. (1 point) Suppose $y_1 = f_1(x)$ and $y_2 = f_2(x)$ are two solutions to the differential equation

$$2y'' + 5y' = y$$

Show that $y = y_1 + 6y_2$ is also a solution.

Since y_1 and y_2 are solutions, this means that

$$\begin{aligned} 2y_1'' + 5y_1' &= y_1 \\ 2y_2'' + 5y_2' &= y_2 \end{aligned}$$

Now if we let $y = y_1 + 6y_2$, we get

$$\begin{aligned} 2y'' + 5y' &= 2(y_1 + 6y_2)'' + 5(y_1 + 6y_2)' \\ &= 2(y_1'' + 6y_2'') + 5(y_1' + 6y_2') \\ &= 2y_1'' + 5y_1' + 6(2y_2'' + y_2') \\ &= y_1 + 6y_2 \\ &= y \end{aligned}$$

so y is also a solution of the differential equation.

(turn over)

3. (1 point) Global Warming! Let $y(t)$ be the number of kilograms of Arctic ice after t days. Suppose that if the air stays at a fixed Fahrenheit temperature T, Arctic ice melts at a rate of $10,000T/y$ kilograms per day. If there are $1,000,000$ kilograms of Arctic ice right now, what is the least temperature T needed for all the Arctic ice to melt within 100 days?

First we have to solve for $y(t)$. From the description of the problem, we see that

$$\frac{dy}{dt} = -\frac{10000T}{y}, \quad y(0) = 1,000,000$$

where the negative sign comes from the fact that the amount of ice is decreasing over time. This differential equation is separable, so we may solve it by moving the y's to one side and the t's to the other, and then integrating.

$$\int y dy = -\int 10^4 T dt$$

$$\frac{y^2}{2} = -10^4 Tt + C$$

Now we use the initial value: $y(0) = 10^6$. So plugging in $t = 0$ and $y = 10^6$, we find $C = 10^{12}/2$. Therefore our solution is

$$\frac{y^2}{2} = -10^4 Tt + \frac{10^{12}}{2}.$$

Now to solve the problem, we have to find an air temperature T such that $y(100) = 0$. So that means:

$$0 = -10^4 T(100) + \frac{10^{12}}{2}$$

so $T = 10^6/2$.