Calculus 1B Quiz 5 Name:

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1. (1 point) Express

$$\frac{x^6}{3-x}$$

as a power series centered at $x = 0$.

We want to use the equation for geometric series: $1/(1-x) = 1 + x + x^2 + \ldots$ so:

$$\begin{aligned}
\frac{x^6}{3-x} &= \frac{x^6}{3} \cdot \frac{1}{1-\frac{x}{3}} \\
&= \frac{x^6}{3}\left(1 + \frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \ldots\right) \\
&= \frac{x^6}{3} + \frac{x^7}{9} + \frac{x^8}{27} + \ldots \\
&= \sum_{n=0}^{\infty} \frac{x^{n+6}}{3^{n+1}}
\end{aligned}$$

2. (1 point) Find the Taylor series of

$$x^4 + 3x$$

centered at $x = 2$.

First:

$$f(x) = x^4 + 3x \quad f'(x) = 4x^3 + 3 \quad f^{(2)}(x) = 12x^2$$
$$f^{(3)}(x) = 24x \quad f^{(4)}(x) = 24 \quad f^{(n)}(x) = 0$$

for all $n \geq 5$. Then

$$f(2) = 22 \quad f^{(1)}(2) = 35 \quad f^{(2)}(2) = 48 \quad f^{(3)}(2) = 48 \quad f^{(4)}(2) = 24$$

and all the other derivatives are 0. So the Taylor series of $f(x)$ at $a = 2$ is:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n = 22 + 35(x-2) + 24(x-2)^2 + 8(x-2)^3 + (x-2)^4$$

(turn over)

3. (1 point) The series

$$\sum_{n=0}^{\infty} \frac{\ln(x)^{n+1}}{(n+1)!}$$

converges for all $x > 0$. What function does this series represent?

(In class hint: What power series in x does this look like? Imagine $\ln(x) = z$.)

Set $z = \ln(x)$. Then the series becomes:

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{z^{n+1}}{(n+1)!} &= \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots \\ &= -1 + \left(1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots\right) \\ &= e^z - 1 = e^{\ln(x)} - 1 = x - 1 \end{aligned}$$

So the series represents $x - 1$.