Calculus 1B Quiz 4 Name:

GSI: Paul Baginski Student ID:

1. Find the interval and radius of convergence of

$$\sum_{n=0}^{\infty} \frac{(x-8)^n}{n!\,3^n}$$

If $x = 8$ then the series converges to 0, so assume $x \neq 8$. Ratio Test.

$$\lim_{n\to\infty} \frac{|x-8|^{n+1}}{(n+1)!\,3^{n+1}} \cdot \frac{n!\,3^n}{|x-8|^n} = \lim_{n\to\infty} \frac{|x-8|}{3(n+1)} = 0$$

So the power series converges for all x. Therefore the interval of convergence is $(-\infty, \infty)$ with radius of convergence ∞.

(turn over)

2. Suppose

$$\sum_{n=0}^{\infty} c_n x^n$$

is a power series with $c_n > 0$ for every n. Its radius of convergence is R with $0 < R < \infty$. If the power series converges at $x = R$, show that it must converge at $x = -R$.

We need to determine that

$$\sum_{n=0}^{\infty} c_n(-R)^n$$

converges. Since $R > 0$ and each $c_n > 0$, this is an alternating series. We can check for absolute convergence. Then $b_n = |c_n(-R)^n| = c_n R^n$, so the series $\sum_{n=0}^{\infty} b_n$ is simply $\sum_{n=0}^{\infty} c_n R^n$, which converges by assumption. So we have absolute convergence and therefore the series for $x = -R$ converges too.

Remark: one might be tempted to use Alternating Series Test here. That *almost* works. We have alternation, and since $\sum_{n=0}^{\infty} c_n R^n$ converges, we have $\lim b_n = 0$. However the b_n do not need to be decreasing and this means AST does not apply. (There actually is a stronger version of AST that does apply here, but that is beyond the scope of the course).