Calculus 1B Quiz 4 Name:

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1. For what x does

$$\sum_{n=0}^{\infty} \left(\frac{x}{3} + 5\right)^n$$

converge? For those x, what does this series converge to?

Set $z = \frac{x}{3} + 5$. Then $3z - 15 = x$ and our power series becomes

$$\sum_{n=0}^{\infty} z^n$$

which is just a plain geometric series. It converges when $|z| < 1$ and diverges for $|z| \geq 1$, so the interval of convergence for z is precisely $(-1, 1)$. Plugging in $z = -1$ and $z = 1$, we get that the interval of convergence for x is $(-18, -12)$, with the radius of convergence for x equal to $|-12 - (-18)|/2 = 3$.

For the values of z when geometric series converges, we know it converges to $1/(1-z)$. The exception is $z = 0$, when it converges to 0. Substituting for x, we get that on the interval of convergence, the power series converges to

$$\frac{1}{1 - \left(\frac{x}{3} - 5\right)} = \frac{-3}{x + 12}$$

except for $x = -15$, in which case it converges to 0.

(turn over)

2. Find the radius and interval of convergence for

$$\sum_{n=0}^{\infty} (-1)^n \frac{\ln(n)}{e^n} x^n$$

Use Ratio Test.

$$\lim_{n\to\infty} \frac{|x|^{n+1}\ln(n+1)}{e^{n+1}} \cdot \frac{e^n}{|x|^n \ln(n)} \quad = \quad \lim_{n\to\infty} \frac{|x|}{e} \cdot \frac{\ln(n+1)}{\ln(n)} \quad =$$

$$\lim_{n\to\infty} \frac{|x|}{e} \cdot \frac{\frac{1}{n+1}}{\frac{1}{n}} \quad = \quad \frac{|x|}{e}$$

So the power series converges for $|x| < e$ and diverges when $|x| > e$.

Now we determine the endpoints. When $x = e$, the power series becomes the series:

$$\sum_{n=0}^{\infty} (-1)^n \ln(n)$$

which diverges by Divergence Test since $\lim a_n \neq 0$ (it does not exist). Similarly, when $x = -e$, we get

$$\sum_{n=0}^{\infty} \ln(n)$$

which diverges by Divergence Test.

So our interval of convergence is $(-e, e)$ and the radius of convergence is e.