Calculus 1B Quiz 1 Name:

GSI: Paul Baginski Student ID:

1. Evaluate: $\int_1^2 x\cos(\pi x)dx$

Set $f = x$ and $g' = \cos(\pi x)dx$. Then $f' = dx$ and $g = \frac{1}{\pi}\sin(\pi x)$. So

$$\begin{aligned}\int_1^2 x\cos(\pi x)dx &= \frac{x}{\pi}\sin(\pi x)\Big|_1^2 - \frac{1}{\pi}\int_1^2 \sin(\pi x)dx \\ = 0 + \frac{1}{\pi^2}\cos(\pi x)\Big|_1^2 &= \frac{1}{\pi^2} - \left(-\frac{1}{\pi^2}\right) = \frac{2}{\pi^2}\end{aligned}$$

2. Integrate: $\int \ln(2x)dx$

Not much choice here. Set $f = \ln(2x)$ and $g' = dx$. Then $f' = \frac{dx}{x}$ and $g = x$. So

$$\int \ln(2x)dx = x\ln(2x) - \int \frac{xdx}{x} = x\ln(2x) - x + C$$

(turn over)

3. Integrate

$$\int x\cot^2(x)dx = \int \frac{x\cos^2(x)}{\sin^2(x)}dx$$

[Hint: f has no denominator.]

Here's a detailed explanation:

The hint tells us that the denominator inside the integral, $\sin^2(x)$, has to be part of g'. In other words,

$$g' = \frac{N(x)}{\sin^2(x)}$$

where the numerator $N(x)$ is a function we still have to determine.

g' is a derivative, so what are possible ways we could take a derivative of a function and get $\sin^2(x)$ in the denominator? One possibility is Quotient Rule, but the Quotient Rule has a pretty complicated numerator and the numerator under the integral is pretty simple (doesn't involve any addition), so this isn't too likely of a possibility. Another way would be Chain Rule on $(\sin(x))^{-1} = 1/\sin(x)$. If you differentiate, you get

$$\left(\frac{1}{\sin(x)}\right)' = \frac{-\cos(x)}{\sin^2(x)}$$

and this looks very similar to something we have in the integral. Let's give it a try: $f = -x\cos(x)$ and $g' = -\cos(x)/\sin^2(x)$. Then $f' = -\cos(x) + x\sin(x)$ and $g = 1/\sin(x)$. This looks promising, since the exponents on the trigonometric functions in f' and g are smaller. Working it out,

$$\int \frac{x\cos^2(x)}{\sin^2(x)}dx \;=\; \frac{-x\cos(x)}{\sin(x)} - \int \frac{-\cos(x) + x\sin(x)}{\sin(x)}dx \;=\; -x\cot(x) + \int \frac{\cos(x)}{\sin(x)}dx - \int x dx$$

and the integrals on the right are all ones we're very familiar with. To finish:

$$\int \frac{x\cos^2(x)}{\sin^2(x)}dx \;=\; -x\cot(x) + \ln(\sin(x)) - \frac{x^2}{2} + C$$

This problem is a good example of when finding a good f and g' is much harder than the computations you do to get the final answer. The reason why this solution looks so long on paper is not because it's a long computation, but because there's a long thought process for finding good f and g'.

If you want more practice, try $\int \cot^2(x)dx$, which follows a similar pattern.