Calculus 1B Quiz 1 Name:

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1. Integrate: $\int t^2 \ln(t) dt$

Set $f = \ln(t)$ and $g' = t^2 dt$. Then $f' = \frac{dt}{t}$ and $g = \frac{1}{3}t^3$. Therefore

$$\int t^2 \ln(t) dt = \frac{1}{3}t^3 \ln(t) - \int \left(\frac{1}{3}t^3\right)\left(\frac{dt}{t}\right) = \frac{1}{3}t^3 \ln(t) - \frac{1}{9}t^3 + C$$

2. Integrate: $\int x^2 \sin(x) dx$

Set $f_1 = x^2$ and $g_1' = \sin(x)dx$. Then $f_1' = 2xdx$ and $g_1 = -\cos(x)$. So

$$\int x^2 \sin(x) dx = -x^2 \cos(x) + 2 \int x \cos(x) dx.$$

Integration by parts again. Set $f_2 = x$ and $g_2' = \cos(x)dx$. Then $f_2' = dx$ and $g_2 = \sin(x)$. So

$$\int x \cos(x) dx = x \sin(x) - \int \sin(x) dx = x \sin(x) + \cos(x) + C, \text{ and so}$$

$$\int x^2 \sin(x) dx = -x^2 \cos(x) + 2\left[x \sin(x) + \cos(x) + C\right] = 2x \sin(x) + 2\cos(x) - x^2 \cos(x) + C$$

(turn over)

3a. Rederive the formula for integration by parts from a well-known formula from Calc 1A.

The product rule for differentiation states that

$$(fg)' = f'g + gf'$$

So $fg' = (fg)' - gf'$. Taking integrals of both sides and using the Fundamental Theorem of Calculus to see $\int (fg)' = fg$, we get

$$\int fg' = fg - \int gf'.$$

3b. Integration by parts lets you integrate $\int fg'dx$ by integrating $\int gf'dx$ instead. Write the formula that integration by parts gives you for $\int fg'dx$. Then write the formula integration by parts gives you for $\int gf'dx$.

$$\int fg' = fg - \int gf'$$
$$\int gf' = gf - \int fg'$$

3c. What happens when you plug the second formula from 3b into the first formula from 3b?

You get the same thing on both sides.

$$\int fg' = fg - \int gf' \;=\; fg - \left[gf - \int fg'\right] \;=\; \int fg'$$

3d. Sometimes to solve an integral, you have to do integration by parts two times. What does 3c. tell you about how to choose your "parts" (f and g') the second time, relative to how you chose the "parts" the first time?

If you choose the same parts the second time, you'll just get back to the integral you started with, without having simplified anything.

In more detail: you first chose f and g' to get:

$$\int fg' = fg - \int gf'$$

and now you have to solve $\int gf'$ by integration by parts. To solve it, you really need to break up gf' into a different combination of functions. Because, if you set $f_2 = g$ and $g_2' = f'$, then $f_2' = g'$ and $g_2 = f$, so integration by parts gives you

$$\int fg' = fg - \int gf' \;=\; fg - \left[f_2 g_2 - \int g_2 f_2'\right] \;=\; fg - \left[gf - \int fg'\right] \;=\; \int fg'.$$