Calculus 1B Midterm Exam 1 Review Sheet GSI: Paul Baginski

For the following sequences, perform these tasks:

(1) Write a general formula for $\{a_n\}$.
(2) Determine if the sequence $\{a_n\}$ converges.
(3) Determine if the series $\sum_{n=1}^{\infty} a_n$ converges.
(4) Determine if the series $\sum_{n=1}^{\infty} a_n$ converges absolutely.
(5) If $\sum_{n=1}^{\infty} a_n$ converges, find an upper and lower bound for the sum.

The sequences:

1. $\frac{5}{2}, \frac{5}{4}, \frac{5}{6}, \frac{5}{8}, \ldots$
2. $\frac{11}{5}, \frac{3}{2}, \frac{19}{15}, \frac{23}{20}, \frac{27}{25}, \frac{31}{30}, \ldots$
3. $\frac{1}{\ln(2)}, \frac{1}{\ln(3)} - \frac{1}{\ln(2)}, \frac{1}{\ln(4)} - \frac{1}{\ln(3)}, \frac{1}{\ln(5)} - \frac{1}{\ln(4)}, \ldots$
4. $\frac{1}{11}, \frac{7}{101}, \frac{25}{1001}, \frac{79}{10001}, \frac{241}{100001}, \ldots$
5. $\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{18}, -\frac{1}{16}, -\frac{\sqrt{2}}{50}, 0, \frac{\sqrt{2}}{98}, \ldots$

For the following integrals, perform these tasks:

(1) Determine if the integral is improper.
(2) If improper, determine if the integral converges.
(3) If improper and convergent, evaluate or find bounds.

The integrals:

1. $\int \sqrt{x} ln(x) dx$
2. $\int_2^3 \frac{x dx}{4x^2-25}$
3. $\int_{-\infty}^{\infty} x e^{-x^2} dx$
4. $\int x \cos(x) \sin(x) dx.$

For the following sequences, determine if the corresponding series converges.

1. $a_n = \frac{\cos(\pi n)}{n^2+9n+8}$

2. $a_n = \frac{3n^4+\sqrt{1+4n^2}}{6n^5\sqrt{n}-n^3+n^2+10}$

3. $a_n = \frac{(4n)!}{(2n+3)!}$

4. $a_n = \frac{2^n}{n^2+n^n}$

5. $a_n = \frac{n}{(\ln(n^n))^2}$ for $n \geq 2$.

For the following series, determine whether the series converges, and if it does, give an upper bound U and a lower bound L for the sum, such that $U - L < \frac{3}{4}$.

1.

$$\sum_{n=1}^{\infty} \left(\frac{\arctan(n)}{\pi} \right)^n$$

2.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$$