REFERENCES: COMPELTELY POSITIVE MAPS

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Here are some references and telegraphic notes on completely positive maps of operator algebras. No attempt has been made to be thorough.

- Vern Paulsen's book [Pau02] is a general reference for basic material on completely positive and completely bounded maps of operator systems.
- Stinespring's theorem is the most basic result on the structure of completely positive maps. It was first proved in [Sti55], in reply to a question of R. V. Kadison about finding a common generalization of the GNS construction and the dilation theorem of Sz.-Nagy [SNF70], originally proved in 1953, concerning dilations of positive operator measures to spectral measures. It was essentially ignored for several years, was reiterated and somewhat simplified in [Arv69], and has now become part of the lore of the subject.
- The extension theorem for completely positive maps was first proved in [Arv69], and subsequent proofs have not changed much (e.g., see [Pau02]). Wittstock's subsequent generalization to completely bounded maps [Wit81] was simplified by Paulsen by essentially reducing it to the extension theorem for completely positive maps, and we will sketch the latter argument from [Pau02].
- The treatment of the Choi-Effros lifting theorem will follow the appendix of [Arv77], after a brief discussion of quasicentral approximate units, which were introduced in the same paper.
- Time permitting, we will also discuss a recent noncommutative generalization of the Perron-Frobenius theorem on positive $n \times n$ matrices to operator algebras [Arv03]. A manuscript is available for downloading from either `http://math.berkeley.edu/~arveson/texfiles.html` or the arXiv.

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