Operator Algebras Seminar

Organizer: W. Arveson

Wednesdays, 2:10–3:30pm, 961 Evans

---

19 Sept. **Bill Arveson**, UC Berkeley

Extremal marginal traces on $M_n \otimes M_n$ and quantum entanglement

M_n denotes the C^*-algebra of all $n \times n$ complex matrices. A *marginal trace* is a state ρ on $M_n \otimes M_n$ with the property $\rho(a \otimes \mathbf{1}) = \rho(\mathbf{1} \otimes a) = \tau(a)$, τ being the tracial state of M_n.

In 2002, Parthasarathy showed that an extremal marginal trace on $M_2 \otimes M_2$ must be a pure (maximally entangled) state of $M_2 \otimes M_2$. Oliver Rudolf has since given an example of an extremal marginal trace of $M_3 \otimes M_3$ that is not pure; earlier, Landau and Streater gave a similar example on $M_4 \otimes M_4$. By analyzing the geometry of the compact space K of marginal traces of rank ≤ 2 on $M_n \otimes M_n$ for $n \geq 4$ in terms of completely positive maps, we show that the extremal ones are generic in the sense that they are an open dense subset of K, and moreover, that all extremals are maximally entangled. More explicitly, if you close your eyes and point to a marginal trace of rank at most 2, you will most likely have a maximally entangled state.

In this first lecture I will discuss basic issues, the so-called separability problem of quantum information theory, and give precise definitions of buzzwords like “maximally entangled states”.