A. Use the divergence theorem to compute $\iint_S (x^3 + y + z^2)\, dS$, where S is the unit sphere $x^2 + y^2 + z^2 = 1$, oriented outwards.

B. Use the divergence theorem to compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle z^2 x, \tan z + \frac{1}{3} y^3, x^2 z + y^2 \rangle$ and S is the upper hemisphere $z = \sqrt{1 - x^2 - y^2}$. Note that S is not closed!

C. Use the divergence theorem to compute $\iint \langle x, y, z \rangle \cdot d\mathbf{S}$, where S is the part of the unit sphere $x^2 + y^2 + z^2 = 1$ where $x \geq 0$, $y \geq 0$, and $z \geq 0$ (oriented outward). Note that S is not closed!

D. Use the divergence theorem to compute the flux of $\langle x^3, y^3, z^3 \rangle$ through the boundary of the solid cylinder $x^2 + y^2 \leq 2$, $0 \leq z \leq 2$.

E. $\mathbf{F} = \left\langle \frac{x}{(x^2+y^2+z^2)^{3/2}}, \frac{y}{(x^2+y^2+z^2)^{3/2}}, \frac{z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$. Compute $\nabla \cdot \mathbf{F}$, and directly compute the flux of $\mathbf{F}$ through the unit sphere. Does this contradict the divergence theorem? Can $\mathbf{F}$ be of the form $\nabla \times \mathbf{G}$?

F. Come up with a two-dimensional version of the divergence theorem. Is it true? Be sure you consider the following points:
1. What is the divergence of a two-dimensional vector field?
2. How would you compute the flux of a vector field through a curve?