Section: 101 113 Name________________________

QUIZ 10 - MATH 53

APRIL 20, 2005

Let $\mathbf{F} = (y^2 z \sec^2 x)\mathbf{i} + (2yz\tan x + 2z)\mathbf{j} + (y^2 \tan x + 2z)\mathbf{k}$, and let C be the curve parameterized by $\mathbf{r}(t) = \langle \sin t, \cos t, t\rangle$, with $0 \le t \le 2\pi$.

1a)[3pts] Write $\mathbf{F}$ as the sum of a conservative vetor field and a simple vector field. The simple vector field should have two coordinates equal to zero.

Computing some partial derivatives and comparing them leads us to believe that $\mathbf{F}$ would be conservative if it weren't for the $2z$ in the $\mathbf{j}$ direction, so we break it off:

$$\mathbf{F} = \nabla(y^2 z \tan x + z^2) + \langle 0, 2z, 0\rangle$$

1b)[3pts] Use part (a) to compute $\int_C \mathbf{F}\cdot d\mathbf{r}$.

We have that

$$\begin{aligned}
\int_C \mathbf{F}\cdot d\mathbf{r} &= \int_C \nabla(y^2 z\tan x + z^2)\cdot d\mathbf{r} + \int_C \langle 0,2,0\rangle \cdot d\mathbf{r} \\
&= ((1)^2(2\pi)\tan(0) + (2\pi)^2) - ((1)^2(0)\tan(0) + (0)^2) + \int_0^{2\pi} \langle 0, 2t, 0\rangle\cdot\langle \cos t, -\sin t, 1\rangle\, dt \\
&= 4\pi^2 - 2\int_0^{2\pi} t\sin t\, dt \\
&= 4\pi^2 - 2(-t\cos t\Big|_0^{2\pi} + \int_0^{2\pi} \cos t\, dt) \\
&= 4\pi^2 + 4\pi
\end{aligned}$$

2)[3pts] Use Green's Theorem to the work done by the vector field $\mathbf{F} = \langle -y^3, x^3\rangle$ on a particle moving counterclockwise around the unit circle.

Green's Theorem tells us that

$$\begin{aligned}
\int_C -y^3\, dx + x^3\, dy &= \iint_D (3x^2 + 3y^2)\, dx\, dy \\
&= 3\int_0^{2\pi}\int_0^1 r^2\, r\, dr\, d\theta \\
&= 3\cdot 2\pi \cdot \frac{1}{4} \\
&= \frac{3\pi}{2}
\end{aligned}$$