CHAPTER 1
The Foundations: Logic, Sets, and Functions

2. **a)** I did not buy a lottery ticket this week.

b) Either I bought a lottery ticket this week or [in the inclusive sense] I won the million dollar jackpot on Friday.

c) If I bought a lottery ticket this week, then I won the million dollar jackpot on Friday.

d) I bought a lottery ticket this week and I won the million dollar jackpot on Friday.

e) I bought a lottery ticket this week if and only if I won the million dollar jackpot on Friday.

f) If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday.

g) I did not buy a lottery ticket this week, and I did not win the million dollar jackpot on Friday.

h) Either I did not buy a lottery ticket this week, or else I did buy one and won the million dollar jackpot on Friday.

4. **a)** If I am to remember to send you the address, then you will have to send me an e-mail message. (This has been slightly reworded so that the tenses make more sense.)

b) If you were born in the United States, then you are a citizen of this country.

c) If you keep your textbook, then it will be a useful reference in your future courses. (The word "then" is understood in English, even if omitted.)

d) If their goaltender plays well, then the Red Wings will win the Stanley Cup.

e) If you get the job, then you had the best credentials.

f) If there is a storm, then the beach erodes.

g) If you log on to the server, then you have a valid password.

6. **a)** This is one-to-one, since if $n_1 - 1 = n_2 - 1$, then $n_1 = n_2$.

b) This is not one-to-one, since, for example, $f(3) = f(-3) = 10$.

c) This is one-to-one, since if $n_1^3 = n_2^3$, then $n_1 = n_2$ (take the cube root of each side).

d) This is not one-to-one, since, for example, $f(3) = f(4) = 2$.

8. **a)** $f(n) = n + 17$ **b)** $f(n) = \lceil n/2 \rceil$

c) We let $f(n) = n - 1$ for even values of n, and $f(n) = n + 1$ for odd values of n. Thus we have $f(1) = 2$, $f(2) = 1$, $f(3) = 4$, $f(4) = 3$, and so on. Note that this is just one function, even though its definition used two formulas, depending on the the parity of n.

d) $f(n) = 17$

10. To clarify the setting, suppose that $g : A \to B$ and $f : B \to C$, so that $f \circ g : A \to C$. We will prove that if $f \circ g$ is one-to-one, then g is also one-to-one, so not only is the answer to the question "yes," but part of the hypothesis is not even needed. Suppose that g were not one-to-one. By definition this means that there are distinct elements a_1 and a_2 in A such that $g(a_1) = g(a_2)$. Then certainly $f(g(a_1)) = f(g(a_2))$, which is the same statement as $(f \circ g)(a_1) = (f \circ g)(a_2)$. By definition this means that $f \circ g$ is not one-to-one, and our proof is complete.