MATH 113, HOMEWORK 6 SOLUTIONS

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Homework #6 (due Tuesday 10/31). p202 #5,6,8,9,10,13 of Fraleigh

#5. False. There is no one $g \in G$ such that gx can be every element of X. Rather, if one lets g *vary*, one can get gx to be any element of X.

#6. A subset of a G-set X is a sub-G-set exactly if X is a union of G-orbits.

#8. a. Not in any natural way, no.

b. Sure. This is required by the identity in G going to the identity permutation in $Sym(X)$.

c. Nope. This just defines the kernel of the homomorphism $G \to Sym(X)$, which was never required to be 1:1 in the definition of G-set.

d. Sure. Apply g^{-1} to both.

e. Nope. This just says $g_2^{-1} g_1 \in Stab(x)$, and that subgroup may have elements other than the identity.

f. Yes.

g. Sure: compose the inclusion $H \to G$ with the homomorphism $G \to Sym(X)$ to get the desired homomorphism $H \to Sym(X)$.

h. Nope. Take for example $G = S_n$, H just the identity, for $n > 1$.

i. Yes.

j. Yes.

#9a. The $\{s_i\}$ and $\{P_i\}$ correspond in an obvious way, and this is D_4-equivariant; all this comes down to is that a rotation does not take the midpoint of an edge off that edge.

b. The left-right flip has two orbits on $\{1, 2, 3, 4\}$ but three orbits on the $\{s_i\}$. So there's no possible D_4-equivariant correspondence between them.

c. Yes. The only remaining candidate pair is the d's vs. the m's, and the same argument (again the left-right flip) as in (b) dispels any correspondence.

10a,b. Yes, D_4 acts faithfully, since it acts faithfully on each of the subsets of size 4. (To check this, see that the stabilizer of an element in each of those – necessarily size 2 – acts faithfully.)

13a. Not much to say here – need to say that if you rotate by α radians, then by β, you do indeed rotate by $\alpha + \beta$ when you're done.

b. A circle centered at the origin. (Note that P was not the origin, by assumption.)

c. The only rotations that bring a non-origin point back to itself are by 2π times an integer. So every G_P is the same, $2\pi\mathbb{Z} \le \mathbb{R}$.

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