Name: Solutions

Quiz 10, Math 1b Sec 201, April 19, 2006

1| (6 pts) Find the general solution to $y' + \ln(x)\, y = \frac{e^x}{x^x}$

We use integrating factors: $I(x) = e^{\int \ln(x)\,dx} = e^{x\ln(x) - x} = x^x e^{-x}$

Then $\frac{d}{dx}\left(I(x)\, y\right) = I(x)\, y' + I(x)\ln(x)\, y = I(x)\frac{e^x}{x^x} = 1$,

So $I(x)\, y = \int 1\, dx = x + C$

$\Rightarrow \quad y = (x + C)\frac{e^x}{x^x} = \frac{e^x}{x^{x-1}} + C\frac{e^x}{x^x}$

2| (5 pts) Find the general solution to $y' + 2xy = x$

Integrating factors again: $I(x) = e^{\int 2x\,dx} = e^{x^2}$

$\Rightarrow \quad \frac{d}{dx}\left(I(x)\, y\right) = e^{x^2} x$

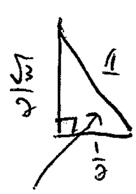
$\Rightarrow \quad I(x)\, y = \int e^{x^2} x\, dx = \frac{1}{2}e^{x^2} + C$

$\Rightarrow \quad y = \frac{1}{2} + Ce^{-x^2}$

3| a) (2pts) Find $\sqrt{i}$

$i = e^{i\frac{\pi}{2}}$

so $\sqrt{i} = (i)^{\frac{1}{2}} = \left(e^{i\frac{\pi}{2}}\right)^{\frac{1}{2}}$



must be 60°

$= e^{i\frac{\pi}{4}}$

$= \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

b) (2 pts) Find $\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^3$

$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$

$\theta = \arctan\left(\frac{\sqrt{3}/2}{-1/2}\right) = \arctan(\sqrt{3}) = \frac{2\pi}{3}$

$\Rightarrow \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^3 = \left(e^{i\frac{2\pi}{3}}\right)^3 = e^{i2\pi} = 1$