MIDTERM # 2 REVIEW WORKSHEET, 11/15/07

MATH 54, FALL 2007

1. Find the best fit line through the points $(-1,0)$, $(0,2)$, $(2,5)$, $(3,6)$. (Hint: Set up a map from the space of lines [i.e. P_1] to $\mathbb{R}^4$ where the coordinates on $\mathbb{R}^4$ correspond to the value of the function at -1, 0, 2, and 3.)

2. (a) Find eigenvalues and eigenvectors for $A = \begin{bmatrix} -4 & -3 \\ 15/2 & 11/2 \end{bmatrix}$. Does A have an eigenbasis?

(b) Find $\lim_{n\to\infty} A^n$, if it exists.

3. (a) Find the eigenvalues of $A = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 1 & 0 \end{bmatrix}$.

(b) Is A diagonalizable?

4. (a) Find an orthogonal basis for the image of $A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \\ 1 & 0 \end{bmatrix}$.

(b) Find the projection of $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ onto the image of A.

5. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} -1 & -4 \\ 1 & -1 \end{bmatrix}$ over $\mathbb{C}$.

6. True or False? Justify your answer.

(a) If $A^T = A$, then any two eigenvectors with distinct associated eigenvalues are orthogonal.

(b) Every matrix has an eigenvector.

(c) If A is symmetric and S is orthogonal, then $S^{-1}AS$ is symmetric.

(d) $\mathrm{im}(AA^T) = \mathrm{im}(A)$

(e) $\det(A+B) = \det(A) + \det(B)$

7. (a) Find all Fourier coefficients of the function

$$f(t) = \begin{cases} 0 & t \leq 0 \\ 1 & t > 0 \end{cases}.$$

(b) Find the projection of $f(t)$ onto the span of 1, $\cos t$, and $\sin t$ (on the interval $[-\pi, \pi]$).