MIDTERM # 1 REVIEW WORKSHEET, 10/3/07

MATH 54, FALL 2007

1. If the image of a linear transformation from P_2 to P_4 is a line, then what's the dimension of the kernel?

2. Find all solutions to the system of equations

$$\left\{ \begin{array}{ccccccc} 2x & + & 2y & + & 2z & = & 8 \\ 2x & + & 3y & + & 3z & = & 1 \\ 3x & + & 4y & + & 4z & = & 5 \end{array} \right\}$$

3. (a) Find the inverse of $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 1 & 2 \end{bmatrix}$.

(b) Solve $A\vec{x} = \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix}$.

(c) Write the matrix $M = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 0 & 5 \end{bmatrix}$ in terms of the basis $\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

4. True or False? Justify your answer.

(a) If W is a subspace of a finite-dimensional linear space V, and we have a basis for W, then we can add vectors to it to get a basis for V.

(b) If W is a subspace of a finite-dimensional linear space V, and we have a basis for V, then we can remove some vectors from it to get a basis for W.

(c) The rank of AB is less than or equal to both the rank of A and the rank of B.

(d) If we change basis, and rewrite the identity matrix in our new basis, we still get the identity matrix.

5. (a) Let $T(f(t)) = f(t^2 - 1) - tf(0)$ as a linear map from P_2 to P_4. Find the matrix for T with respect to the bases $(1, t, t^2)$ on P_2 and $(1, t, t^2, t^3, t^4)$ on P_4.

(b) Find bases for the kernel and image of T.

6. If we write the matrix $M = \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & & | \end{bmatrix}$ in the basis $(\vec{v}_1, \ldots, \vec{v}_n)$, what do we get?

7. (a) Suppose $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ is a non-zero $n \times 1$ matrix and $\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \ldots & v_m \end{bmatrix}$ is a non-zero $1 \times m$ matrix. Let $A = \mathbf{uv}$, an $n \times m$ matrix. Show that A has rank 1.

(b) Conversely, show that if A is an $n \times m$ matrix of rank 1, then there exist $\mathbf{u}$ (an $n \times 1$ matrix) and $\mathbf{v}$ (a $1 \times m$ matrix) such that $A = \mathbf{uv}$.