REVIEW WORKSHEET FOR FINAL, 12/17/07

MATH 54, FALL 2007

1. Find all solutions to $\mathbf{x}' = A\mathbf{x}$ for

(a) $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 10 \\ -1 & 0 \end{bmatrix}$

2. Find e^A for the A in 1(a).

3. Solve

$$\frac{\partial u}{\partial t} = 9\frac{\partial^2 u}{\partial x^2} \quad (1)$$

$$u(0,t) = 2 \quad (2)$$

$$u(1,t) = 0 \quad (3)$$

$$u(x,0) = 0 \quad (4)$$

4. (a) Let $T(f(t)) = f'(t) + tf(2t)$ as a linear map from P_2 to P_3. Find the matrix $[T]_\gamma^\beta$ for T with respect to the bases $\beta = (1, t, t^2)$ on P_2 and $\gamma = (1, t, t^2, t^3)$ on P_4.

(b) What's the rank of T? The nullity? Write out the rank-nullity formula for T.

(c) Find bases for the kernel and image of T.

5. (a) What's the dimension of the subspace V of $\mathbb{R}^3$ defined by $x_1 + x_2 + 2x_3 = 0$?

(b) Find the projection of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ onto V.

(c) Find the matrix for orthogonal projection onto V.

6. True/False

(a) The matrix A in 1(a) is diagonalizable.

(b) The matrix A in 1(b) is diagonalizable.

(c) $\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}$ is an orthogonal matrix.

(d) The kernel of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$ is isomorphic to P_2.

7. Find the inverse of the matrix A in 1(a).