WORKSHEET #8, 9/20/07

MATH 54, FALL 2007

1. Find a basis of the image of $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

2. Suppose the column vectors of a matrix A are (in order) v_1, v_2, and v_3. If $A \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \overrightarrow{0}$, show that the column vectors are linearly dependent and exhibit a linear dependence (i.e. write one of them in terms of the others).

3. True or False? Justify your answer. If $v_1, \ldots v_n$ is a basis of $\mathbb{R}^n$ and A is an invertible $n \times n$ matrix, then $Av_1, \ldots, Av_n$ is also a basis of $\mathbb{R}^n$.

4. (Repeated from last worksheet.) (a) Let A be the 2×2 matrix for projection to the x-axis and let B be the 2×2 matrix for rotation counterclockwise by $\pi/2 = 90°$. Write down A and B.

(b) If we first project to the x-axis and then rotate by 90° counterclockwise, describe what happens geometrically (i.e. draw a picture or say what happens to $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$).

(c) Repeat (b), but first rotate and then project. Did you get the same answer?

(d) Find AB and BA. Are they the same? Which one corresponds to (b) and which one corresponds to (c)?

(e) Find vector(s) which span the kernel of A. Do the same for B, AB, and BA. Are some the same?

(f) Show that if we have two matrices C and D and D is invertible, then $\ker(DC) = \ker(C)$.

(g) Also show that $\ker(CD) = D^{-1}(\ker(C))$. (This last bit means "the set of all vectors of the form $D^{-1}\overrightarrow{v}$ for $\overrightarrow{v}$ in $\ker(C)$.") Check that this was the case for A and B and ponder this geometrically.

(h) Find vector(s) which span the image of A. Do the same for B, AB, and BA. Are some the same?

(i) Again with C and D with D invertible, show: (1) $\text{im}(CD) = \text{im}(C)$ and (2) $\text{im}(DC) = D(\text{im}(C))$ (note that (2) doesn't require D invertible). Check that this was the case for A and B and ponder this geometrically.