WORKSHEET #7, 9/18/07

MATH 54, FALL 2007

1. Write down a matrix whose image is the span of $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$.

2. (a) Find vector(s) spanning the kernel of $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & -1 & 0 \end{bmatrix}$.

(b) Find vector(s) spanning the image of A. If you can, find a minimal set of vectors spanning the image of A (i.e. no redundant vectors).

3. (a) Let A be the 2×2 matrix for projection to the x-axis and let B be the 2×2 matrix for rotation counterclockwise by $\pi/2 = 90°$. Write down A and B.

(b) If we first project to the x-axis and then rotate by 90° counterclockwise, describe what happens geometrically (i.e. draw a picture or say what happens to $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$).

(c) Repeat (b), but first rotate and then project. Did you get the same answer?

(d) Find AB and BA. Are they the same? Which one corresponds to (b) and which one corresponds to (c)?

(e) Find vector(s) which span the kernel of A. Do the same for B, AB, and BA. Are some the same?

(f) Show that if we have two matrices C and D and D is invertible, then $\ker(DC) = \ker(C)$.

(g) Also show that $\ker(CD) = D^{-1}(\ker(C))$. (This last bit means "the set of all vectors of the form $D^{-1}\overrightarrow{v}$ for $\overrightarrow{v}$ in $\ker(C)$.) Check that this was the case for A and B and ponder this geometrically.

(h) Find vector(s) which span the image of A. Do the same for B, AB, and BA. Are some the same?

(i) Again with C and D with D invertible, show: (1) $\mathrm{im}(CD) = \mathrm{im}(C)$ and (2) $\mathrm{im}(DC) = D(\mathrm{im}(C))$ (note that (2) doesn't require D invertible). Check that this was the case for A and B and ponder this geometrically.

4. Consider the space $\mathbb{R}^3$ (i.e. "three-dimensional space", or "vectors with three coordinates"). For each of the following, decide whether the set described is a subspace of $\mathbb{R}^3$ or not.

(a) All of $\mathbb{R}^3$.

(b) The set $\{\overrightarrow{0}\}$ (i.e. the set containing only the zero vector).

(c) The set ϕ, i.e. the empty set (the set with no elements).

(d) The line $\left\{ \begin{bmatrix} 2t \\ 3t \\ 0 \end{bmatrix} : t \text{ is any real number} \right\}$

(e) The plane $x_1 + x_2 = 0$.

(f) The plane $x_1 + x_2 = 5$.

(g) The span of the vectors $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$.