WORKSHEET #3, 9/4/07

MATH 54, FALL 2007

1. (a) Write down the transformation from $\mathbb{R}^2$ to itself which is given by reflection about the x-axis (i.e. write $x_{new} =$ (something in terms of x_{old} and y_{old}) and similarly for y_{new}).

(b) Is the transformation you wrote down in part (a) linear? If so, write down its matrix.

2. (a) and (b) Repeat problem one for the transformation from $\mathbb{R}^2$ to itself given by the "shear" which is described by "a vector stays at the same height (i.e. y-value) and is moved to the right by a quantity equal to its height (note: this means it's actually moved to the left if its height is negative)."

(c) Draw some vectors and where they go under this transformation. Can you see why the term "shear" is appropriate?

3. Find the inverse of $\begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$.

4. (cf §2.1 problem 13) Show that the 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if and only if $ad - bc \neq 0$, in which case its inverse is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

[Hint: Consider the case $a \neq 0$ first and go through the process of finding its inverse (but check whether you get stuck when $ad - bc = 0$) and then do the same for the case $a = 0$.]

Note 1: There are formulas for larger matrices too, but they're not nearly this nice.

Note 2: Just memorizing this formula is probably not OK on quizzes and tests. (Also, you'll need to know how to do larger matrices.)

Note 3: The quantity $ad - bc$ is called the *determinant* of A (for 2×2 matrices). It and its definition for larger matrices will show up later on.