

# WORKSHEET #3, 9/4/07

MATH 54, FALL 2007

1. Compute the following:

$$(a) \begin{bmatrix} 3 & -1 & -1 \\ 2 & 1 & 0 \\ 1 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 4 \\ 1 & 1 & 1 \\ -1 & -2 & -3 \end{bmatrix}$$

$$(b) \begin{bmatrix} 3 & -1 & -1 \\ 2 & 1 & 0 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

2. Put each of the following matrices into reduced row-echelon form and find their ranks.

$$(a) A = \begin{bmatrix} 1 & 8 & 3 \\ 1 & 9 & 4 \\ 0 & 2 & 3 \end{bmatrix}$$

$$(b) B = \begin{bmatrix} 1 & 8 & 3 \\ 1 & 9 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

3. For the two matrices  $A$  and  $B$  in problem 2, answer the following:

$$(\text{note that } \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} )$$

(a) Can the system of equations  $A\vec{x} = \vec{b}$  ever be inconsistent? How about  $B\vec{x} = \vec{b}$ ?

(b) Can the system of equations  $A\vec{x} = \vec{b}$  ever have infinitely many solutions? How about  $B\vec{x} = \vec{b}$ ?

(c) Can the system of equations  $A\vec{x} = \vec{b}$  ever have a unique solution? How about  $B\vec{x} = \vec{b}$ ?

(d) (Challenge) For each of the six questions above, if you answered yes, try to find an example of a vector  $\vec{b}$  satisfying the conditions.