WORKSHEET #29, 12/6/07

MATH 54, FALL 2007

0. Find the Fourier sine series for $f(x) = x$ on the interval $[0, \pi]$.

1. Solve the heat equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ with the given boundary conditions. (Your answer for some of the following may be an infinite sum.)

(a) $u(0,t) = 0, u(\pi,t) = 0$ and $u(x,0) = x$.

(b) $u(0,t) = 1, u(\pi,t) = 1$ and $u(x,0) = 1$.

(c) $u(0,t) = 0, u(\pi,t) = \pi$ and $u(x,0) = x$.

(d) $u(0,t) = 0, u(\pi,t) = 1$ and $u(x,0) = 1$.

2. (Repeated from last worksheet.) Consider the wave equation boundary value problem (but don't solve it yet)

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} \tag{1}$$

$$u(0,t) = 0 \tag{2}$$

$$u(1,t) = 0 \tag{3}$$

$$u(x,0) = 3\sin \pi x - 2\sin 4\pi x \tag{4}$$

$$\frac{\partial u}{\partial t}(x,0) = \sin \pi x + 4\sin 7\pi x \tag{5}$$

Find the general solution to (1)-(3).

(e) Impose condition (4). What does this tell you about some of the constants?

(f) Impose also condition (5). What does this tell you about the rest of the constants? What is the final answer $u(x,t)$?

3. (a) Look at 1(a) and 1(d). Does it look like there's something wrong with the boundary conditions?

(b) There are several explanations of this. I should note that the problems do make sense if interpreted properly and that for $t > 0$ the solution $u(x,t)$ is smooth (i.e. continuous and infinitely differentiable). Looking at the equation (or thinking physically about heat), can you see[1] why, if you have a discontinuity at time $t = 0$ in your distribution of heat, that it would instantly be smoothed out (i.e. be smoothed out at any future time $t > 0$)?

[1] non-rigorously