

## WORKSHEET #28, 12/4/07

MATH 54, FALL 2007

1. Consider the heat equation boundary value problem (but don't solve it yet)

$$\begin{aligned}(1) \quad & \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2} \\(2) \quad & u(0, t) = 0 \\(3) \quad & u(1, t) = 0 \\(4) \quad & u(x, 0) = 3 \sin \pi x - 2 \sin 4\pi x\end{aligned}$$

(a) Find all solutions to (1) given by separation of variables  $u(x, t) = X(x)T(t)$ . (If you like you may assume that the constant  $K$  you get is negative, as the other solutions will not contribute to this type of problem.)

(b) Impose the condition (2) to your solution  $X(x)T(t)$ . Which are possible now?

(c) Also impose the condition (3) to your solution  $X(x)T(t)$ . Which are possible now? [Note: Here is where you'd be able to rule out  $K \geq 0$ .]

(d) Write  $u = \sum c_n u_n$  where  $u_n$  are the solutions you found in (c) and the  $c_n$  are constants. Is this also a solution to (1)-(3)?

(e) Impose condition (4) on what you found in (d). What do you find for the  $c_n$ 's? What is the final answer  $u(x, t)$ ?

2. (a) Find the Fourier coefficients for  $f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$  on the interval  $[-1, 1]$ .

(b) Write the Fourier series for  $f(x)$ . That is, write  $f(x)$  as an infinite sum of multiples of 1, sines, and cosines.

(c) Consider (1)-(3) from problem 1 and the (general) solution you found in 1(c). Use (instead of (4))  $u(x, 0) = f(x)$ . What are the  $a_n$ 's from 1(d) now? What's the (formal) solution to this heat flow problem.

3. Consider the wave equation boundary value problem (but don't solve it yet)

$$\begin{aligned}(5) \quad & \frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2} \\(6) \quad & u(0, t) = 0 \\(7) \quad & u(1, t) = 0 \\(8) \quad & u(x, 0) = 3 \sin \pi x - 2 \sin 4\pi x \\(9) \quad & \frac{\partial u}{\partial t} = \sin \pi x + 4 \sin 7\pi x\end{aligned}$$

Repeat (a)-(d) from problem 1 for this setup.

(e) Impose condition (8). What does this tell you about some of the constants?

(f) Impose also condition (9). What does this tell you about the rest of the constants? What is the final answer  $u(x, t)$ ?