WORKSHEET #27, 11/29/07

MATH 54, FALL 2007

1. Determine all solutions to the boundary value problems:

(a) $y'' + 4y' + 3y = 0$, $y(0) = 0$, $y(1) = 0$.

(b) $y'' + y = 0$, $y(0) = 0$, $y(\pi) = 0$.

(c) $y'' + 4y = 0$, $y(0) = 0$, $y(\pi/2) = 1$.

(d) $y'' + y = 0$, $y'(0) = 0$, $y(2\pi) = 1$.

2. Find all values of λ for which the given boundary value problem has a solution:

(a) $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$.

(b) $y'' + \lambda y = 0$, $y'(0) = 0$, $y(1) = 0$.

3. Suppose you have a string pulled reasonably tight and attached at its two ends 1 meter apart, at $x = 0$ and $x = 1$. Suppose the string only vibrates up and down (i.e. not left and right). Let $y(x, t)$ denote the height of the string at position x and time t.

(a) We'll probably discuss soon why this satisfies the differential equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$. Suppose $y(x, t)$ is of the form $X(x)T(t)$. Plug this into both sides and get an eigenvalue problem for $X(x)$.

(b) For what constants is your eigenvalue problem solvable?

(c) Describe physically what this has to do with the string being held down at its endpoints.