WORKSHEET #22, 11/8/07

MATH 54, FALL 2007

1. Find the general solution to $y''' - y = 0$.

2. Rewrite $t^3 y''(t) + \cos(t)y'(t) + 2y(t) = e^t$ as a first-order system in the normal form $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$.

3. (**Euler's method**) Let's find an approximate solution to the equation $y''(t) - 2y'(t) + y(t) = t^2$ given the initial conditions $y(0) = 1$, $y'(0) = 2$. (We can solve this exactly as well, but don't do that for now.)

(a) Find $y''(0)$.

(b) Use $y(t) \approx y(0) + ty'(0)$ for t near zero to approximate $y(.1)$.

Use $y'(t) \approx y'(0) + ty''(0)$ for t near zero to approximate $y'(.1)$.

(c) Find $y''(.1)$ (approximately). Repeat part (b) but using $t = .1$ as your initial value and $t = .2$ as your ending value. Note that you can continue repeating this process.

(d) Was it necessary to start with an initial value for both y and y' in order to get an approximation for $y(t)$? Does this method provide a sort of explanation for why you get a unique solution given initial values for $y(0)$ and $y'(0)$ for a second order equation?

(e) Would the above process have worked if the equation were $ty''(t) - 2y'(t) + y(t) = t^2$?

(f) Write the original system as a first-order system in normal form. What are the initial conditions? Can you carry out Euler's method in this context? Is it similar to the above (i.e. a-c)?

4. Decide whether the given vector-valued functions are linearly independent.

(a) $\begin{bmatrix} e^{2t} \\ 1 \end{bmatrix}, \begin{bmatrix} e^{t} \\ e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} 2e^{t} \\ -4\sin(t) \end{bmatrix}, \begin{bmatrix} -3e^{t} \\ 6\sin(t) \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ \sin(t) \end{bmatrix}, \begin{bmatrix} t \\ 4 \end{bmatrix}, \begin{bmatrix} t^2 \\ \sin(t) \end{bmatrix}$