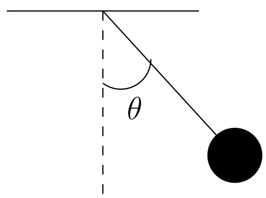
WORKSHEET #22, 11/8/07

MATH 54, FALL 2007

1. Find the general solution to $y'' - 4y' + 13y = 0$.

2. The figure shows a pendulum with length L and angle θ from the vertical. Using just Newton's Law $F = ma$ one can show (skip this step but try it at home if you like) that the angle $\theta(t)$, as a function of time, satisfies $\theta'' + \frac{g}{L}\sin\theta = 0$, where g is the acceleration due to gravity (i.e. a constant).



a) This equation isn't linear. Use the first-order approximation to $\sin\theta$ (coming from its Taylor expansion, i.e. $\sin\theta \approx \theta$) to make the differential equation linear. This approximation will be valid when the angle θ is small.

Note: Nonlinear differential equations are in general difficult. Usually one tries to understand a linear approximation before (or instead of) attempting to understand any nonlinear differential equation.

b) Find the general solution for [your approximation of] the equation of motion of the pendulum (i.e. find the general solution of your differential equation from part (a)).

c) What is the frequency of motion of the pendulum? What does it depend on (mass? length? initial position or velocity?)?

d) If you want the pendulum on your grandfather clock to swing back and forth every two seconds (why two? "tick, tock"), how long should the pendulum be? (Use $g = 9.8$ m/s^2. Use a calculator or just roughly estimate.) Does your answer agree with how long you think the pendulum on a grandfather clock usually is?

3. (a) Use the Wronskian to check that $\cos x$, $\sin x$, $\cos(2x)$, and $\sin(2x)$ are linearly independent. (Hint: You only need to show that the Wronskian is nonzero at one point. Choose an easy point to test it at.)

(b) Can you think of any other way to show they're linearly independent? (I can think of one using the basics of Fourier Series that we learned a little while ago.)

4. Describe the set of all functions whose n^{th} derivative is zero. (Hint: Set up differential equation and find all solutions to it.)

5. (a) Construct a linear differential operator L from C^∞ to C^∞ which has $e^{r_1 x}, e^{r_2 x}, \ldots e^{r_n x}$ in its kernel but does not have e^{rx} in its kernel for any other r.

(b) Argue that this implies that there can be no linear dependence involving only functions of the form e^{rx} for distinct r's.