# WORKSHEET #18, 10/25/07

MATH 54, FALL 2007

1. (a) Consider a transformation $T$ from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by reflection about a line $L$ in $\mathbb{R}^2$. What are its eigenvalues?

(b) Can you find a $2 \times 2$ matrix with those same eigenvalues which is not a reflection about a line?

2. We can use the theory of eigenvectors and eigenvalues to get an explicit formula for the $n^{\text{th}}$ Fibonacci number. Recall that the Fibonacci numbers are 0, 1, 1, 2, 3, 5, 8, 13, 21, 35, . . . . Each number is the sum of the two preceding; that is, $F_0 = 0$ and $F_1 = 1$ and then $F_{n+1} = F_{n-1} + F_n$.

(a) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Check that $A \begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix} = \begin{bmatrix} F_{n+1} \\ F_n \end{bmatrix}$.

Check that this means that $A^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} F_{n+1} \\ F_n \end{bmatrix}$.

(b) (Find the eigenvalues and eigenvectors for $A$ if you already know how. If not, I'll tell you the eigenvectors and you can figure out the eigenvalues:) Let $\phi$ denote the *golden ratio*. That is, $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$ is the positive solution to $\phi^2 - \phi - 1 = 0$. It is the unique positive real number which satisfies $\frac{1}{\phi} = \phi - 1$. Note that it also satisfies $\phi + 1 = \phi^2$.

Check that $v_1 = \begin{bmatrix} \phi \\ 1 \end{bmatrix}$ is an eigenvector of $A$. What is its eigenvalue $\lambda_1$? Check that $v_2 = \begin{bmatrix} 1 \\ -\phi \end{bmatrix}$ is an eigenvector of $A$. What is its eigenvalue $\lambda_2$?

(c) Write $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as a linear combination of $v_1$ and $v_2$.

(d) Find $A^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ explicitly in terms of $v_1$ and $v_2$.

Use this together with (a) to find an explicit formula for $F_n$.