WORKSHEET #13, 10/9/07

MATH 54, FALL 2007

1. Perform the Gram-Schmidt process on the following basis for $\mathbb{R}^3$: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$. [Note: if you think you'll get to problem 5, remember to save your work for this problem.]

2. Sketch two random linearly-independent, non-orthogonal, non-unit [make up your own units] vectors in $\mathbb{R}^2$. "Perform" the Gram-Schmidt process on these and make a sketch of the resulting vectors.

3. Find an orthonormal basis of the kernel of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$.

4. Recall the law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\theta$, where a, b, and c are the lengths of the sides of a triangle and θ is the angle opposite the side of length c.

Sketch a triangle with sides given by the vectors $\overrightarrow{x}$, $\overrightarrow{y}$, and $\overrightarrow{x} - \overrightarrow{y}$. Expand $||\overrightarrow{x} - \overrightarrow{y}||^2 = (\overrightarrow{x} - \overrightarrow{y}) \cdot (\overrightarrow{x} - \overrightarrow{y})$ and use the law of cosines to prove that $\overrightarrow{x} \cdot \overrightarrow{y} = ||\overrightarrow{x}||\,||\overrightarrow{y}||\cos\theta$.

5. Let $M = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ (cf. problem 1). Use your work from problem 1 to find a QR-factorization for M. That is, write M as a product of an orthogonal matrix and an upper triangular matrix. (Hint: the Q matrix is the matrix whose columns are the orthonormal basis vectors you found, and the R matrix is a change-of-basis matrix.)