WORKSHEET #12, 10/4/07

MATH 54, FALL 2007

0. Find a basis for the kernel of $\begin{bmatrix} 0 & 0 & 1 & 2 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

1. (a) Find a vector of length one parallel to $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

(b) Find another. Are there more than these two?

(c) Project $\begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ onto the line spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

(d) Does it matter whether you use the vector you found in (a) or (b) (in the projection formula)?

(e) What is the angle between the two vectors mentioned in part (c)? (This isn't all that related to the rest of the problem.)

2. (a) Check that $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$ are orthogonal. (Are they orthonormal?)

(b) Project $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ onto the plane spanned by v_1 and v_2.

3. (a) Argue geometrically that $||x + y|| \leq ||x|| + ||y||$. This is known as the Triangle Inequality.

(b) Use the Triangle Inequality to prove the Cauchy-Schwarz inequality $|x \cdot y| \leq ||x||||y||$. (Hint: Square both sides of the Triangle inequality and use the fact that $||x + y||^2 = (x + y) \cdot (x + y)$.

(c) Really one should take the opposite approach: the book proves the Cauchy-Schwarz inequality algebraically but not the Triangle Inequality. Use the Cauchy-Schwarz inequality to prove (algebraically) the Triangle Inequality. (Hint: Again use $||x + y||^2 = (x + y) \cdot (x + y)$. Essentially you want to make the argument you made in part (b) in reverse.)

(d) Prove the Parallelogram Equality: $||x+y||^2 + ||x-y||^2 = 2(||x||^2 + ||y||^2)$ (algebraically). Draw a picture to see why it's called what it is.