WORKSHEET #11, 10/2/07

MATH 54, FALL 2007

1. Find matrices for the following linear transformations in the given bases:

(a) $T(f) = \int_0^1 f(x)\,dx$ from P_2 (polynomials of degree ≤ 2) to $\mathbb{R}$, with respect to the basis $(1, x, x^2)$ on P_2 and the standard basis on $\mathbb{R}$. If you like, find the kernel of T.

(b) $T(M) = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} M$ from $\mathbb{R}^{2\times 2}$ (i.e. 2×2 matrices) to itself, with respect to the usual basis (i.e. the four matrices with a one in one spot and zeroes elsewhere). Is this an isomorphism?

2. Find a polynomial of degree 2 passing through the points $(1, -2)$, $(2, 0)$, and $(4, 10)$. If you're not sure where to start, follow these steps:

(a) Consider the linear transformation $T(f) = (f(1), f(2), f(4))$ from P_2 to $\mathbb{R}^3$. Find its matrix with respect to the usual bases.

(b) Find a polynomial f such that $T(f) = (-2, 0, 10)$. (Is this the polynomial you were looking for?)

Note: This method can be used to find a polynomial of degree n passing through any $n + 1$ given points (with different x values). The idea here is similar to (but not exactly the same as) how computers draw curves passing through specified points (for example in a drawing program). See also problem 4.3.69 in the book (which you had for homework).

3. (a) Let $T(f) = (f(0), f'(0), f''(0), f'''(0))$ from P_3 to $\mathbb{R}^4$. Find a matrix for this with respect to the usual bases. Furthermore, show that this is an isomorphism.

(b) Consider the map in problem 35 on your homework, that is $T(f) = (f(0), f'(0), f''(0), f'''(0), \ldots)$ from P (all polynomials) to V (infinite sequences of real numbers). Show that it is injective.

(c) Show that it is not surjective. (Can you find an explicit sequence which is not in the image?)

(d) Let $N \subset V$ be the subspace consisting of infinite sequences of real numbers which are eventually zero (i.e. the sequence is $0, 0, 0, \ldots$ past some point). Show that T gives an isomorphism from P to N.