WORKSHEET #10, 9/27/07

MATH 54, FALL 2007

1. (a) Write the transformation given in the standard basis by the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ in terms of the basis $\vec{v}_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

(b) Make a sketch of the coordinate grid for the basis $\vec{v}_1$, $\vec{v}_2$ and sketch $A\vec{v}_1$ and $A\vec{v}_2$ on it. Visually verify your answer to part (a).

2. Which of the following are subspaces? For each that is a subspace, what is its dimension? If furthermore the dimension is finite, find a basis.

(a) The space of all diagonal 3×3 matrices (inside the linear space $\mathbb{R}^{3\times 3}$ of all 3×3 matrices).

(b) The space of all polynomials $p(x)$ of degree at most 3 such that $p''(3) = 0$ (inside the linear space P_3 of all polynomials of degree at most 3).

(c) The space of polynomials of degree exactly 3 (inside the linear space P_3 of all polynomials of degree at most 3).

(d) The space of functions $f(t)$ from $\mathbb{R}$ to $\mathbb{R}$ for which $f(0) \neq 1$ (inside the linear space $F(\mathbb{R}, \mathbb{R})$ of all functions from $\mathbb{R}$ to $\mathbb{R}$).

(e) The space of differentiable functions $f(t)$ from $\mathbb{R}$ to $\mathbb{R}$ for which $f'(0) = f'(1) = 0$ (inside the linear space $C^1(\mathbb{R})$ of all differentiable functions from $\mathbb{R}$ to $\mathbb{R}$).

(f) The space of differentiable functions $f(t)$ from $\mathbb{R}$ to $\mathbb{R}$ which solve the differential equation $f'(t) = f(t)$ (inside the linear space $C^1(\mathbb{R})$ of all differentiable functions from $\mathbb{R}$ to $\mathbb{R}$).

3. We say that two matrices A and B are *similar* if there's an invertible matrix S such that $S^{-1}AS = B$ (that is, if B can be thought of as the same linear transformation but in a different basis, given by the change of basis matrix S).

(a) Show that reflection about a line L in $\mathbb{R}^2$ is similar to $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (hint: choose a convenient basis).

(b) Show that if two matrices are similar, then they have the same rank.

(c) A *similarity class* is a collection of matrices which are all similar to each other.[1] Consider all 2×2 matrices from which are of rank 1. Show that they are *not* all similar to each other (that is, find two such which are not similar; this shows that they are not all in the same similarity class).

(d) Challenge: Find all similarity classes of 2×2 matrices of rank 1 (by finding a representative for each one).

[1] More precisely, I should require that the collection includes at least one matrix and also includes all matrices which are similar to any matrix in the collection [which is equivalent to just taking all the matrices similar to a given one].