QUIZ SOLUTIONS #9, 9/25/07

MATH 54, FALL 2007

Show your work and justify your answers! Feel free to use both sides.

Name:

1. (4 pts) Write $\vec{x} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ in terms of the basis $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. (That is, write $\vec{x}$ as a linear combination of the v's).

We could do this by inspection (i.e. "ad hoc"), but let's do it systematically. We want to find c's such that $\vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$, which is the same as solving:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}.$$

To do this we set up an augmented matrix as usual, and solve (but our "variables" happen to be called c_1, c_2, c_3 — don't be intimidated by this). I'll skip the rest, but we get $c_1 = 1$, $c_2 = 1$, and $c_3 = 3$. It's quite easy to check our answer, so be sure to do that in this sort of problem.

2. (3 pts each) Find the dimension. Be sure to justify your answers.

(a) What's the dimension of the subspace in $\mathbb{R}^4$ defined by solutions to the equation $x_1 + 3x_2 - 3x_3 + x_4 = 0$?

This subspace is the kernel of the matrix $\begin{bmatrix} 1 & 3 & -3 & 1 \end{bmatrix}$, which I'll call A. We can do this two ways.
First method: Use rank-nullity, $\dim(\text{im}(A)) + \dim(\ker(A)) = \#$ of columns ($= \dim(\text{domain})$). There are 4 columns, and the dimension of the image is the rank, which is 1, so we get $1 + \dim(\ker(A)) = 4$, so $\dim(\ker(A)) = 3$, so the solutions form a 3-dimensional subspace of $\mathbb{R}^4$.
Second method: When we solve the system of equations, we'll get as many basis vectors for the kernel as there are free columns (i.e. one for each parameter), and there are 3 of those, so the solutions form a 3-dimensional subspace of $\mathbb{R}^4$.

(b) A linear transformation from $\mathbb{R}^5$ to $\mathbb{R}^8$ has image a line (i.e. spanned by one vector). What's the dimension of its kernel?

We use rank-nullity: $\dim(\text{im}(A)) + \dim(\ker(A)) = \#$ of columns ($= \dim(\text{domain})$). Here the domain is $\mathbb{R}^5$, which is 5-dimensional.
The image is a line, which is 1-dimensional.
Thus $1 + \dim(\ker(A)) = 5$, so $\dim(\ker(A)) = 4$.