QUIZ SOLUTIONS #8, 9/20/07

MATH 54, FALL 2007

Show your work and justify your answers! Feel free to use both sides.

Name:

1. (4 pts) Find a basis of the image of $A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 2 & 2 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$.

The first column vector is nonzero, so we keep it. The second column vector is not a multiple of the first (for example, it has nonzero entries in places the first has zero entries). The third is the sum of the first two, so we throw it out (i.e. it's redundant). The last is the same as the second, so it's redundant too and we throw it out. Thus a basis for the image of A is:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

2. (2 pts each) True or False? Briefly justify your answers.

(a) The horizontal line in $\mathbb{R}^2$ given by $x = 1$ is a subspace of $\mathbb{R}^2$.

FALSE. For example, it doesn't contain $\vec{0}$.

(b) Any three vectors in $\mathbb{R}^3$ (i.e. three-space) span $\mathbb{R}^3$.

FALSE. They may span only a proper subspace. That is, they may span only a plane, or only a line, or even (if they're all zero) only the zero vector.

(c) The column vectors of a matrix A are linearly independent if and only if $\ker(A) = 0$.

TRUE. Linear dependences are in one-to-one correspondence with nonzero elements of the kernel:

$$A = \begin{bmatrix} | & \cdots & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & \cdots & | \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \vec{0}$$

is the same as

$$c_1 \vec{v}_1 + \ldots + c_n \vec{v}_n = \vec{0},$$

which is a dependence when not all of the c_i's are zero.