QUIZ SOLUTIONS #7, 9/18/07

MATH 54, FALL 2007

1. (3 pts) Find the product $\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

2. (a) (4 pts) Find vector(s) which span the kernel of $A = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 6 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -3 & -9 & 0 & -3 \end{bmatrix}$.

We solve $A\overrightarrow{x} = \overrightarrow{0}$:

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 2 & 6 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -3 & -9 & 0 & -3 & 0 \end{array}\right] \begin{array}{l} \\ -2I \\ \\ +3I \end{array} \leftrightarrow \left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Thus x_1 is a dependent variable and x_2, x_3, and x_4 are free variables. We let $x_2 = r$, $x_3 = s$, and $x_4 = t$. Then $x_1 = -3r - t$. Thus

$$\ker(A) = \left\{ \begin{bmatrix} -3r - t \\ r \\ s \\ t \end{bmatrix} : r, s, t \text{ are any real numbers} \right\}.$$

Thus

$$\ker(A) = \text{span}\left(\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right).$$

(b) (3 pts) Describe the image of A (for example by finding vector(s) which span it).

The image is the span of the column vectors. That is,

$$\text{im}(A) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 0 \\ -9 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix} \right).$$

The last three are redundant, so we may also write (this wasn't required):

$$\text{im}(A) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix} \right).$$

That is $\text{im}(A)$ is a line spanned by the above vector.