QUIZ #6, 9/13/07

MATH 54, FALL 2007

Show your work and justify your answers! Feel free to use both sides.

Name:

1. (4 pts) Find the inverse to $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 3 & 3 & 3 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ \\ -3I \end{matrix} \quad \leftrightarrow \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & -3 & 0 & 1 \end{array}\right] \begin{matrix} -II \\ \\ \\ \end{matrix} \quad \leftrightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & -3 & 0 & 1 \end{array}\right] \begin{matrix} \\ \\ \times\frac{1}{3} \end{matrix} \quad \leftrightarrow \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & \frac{1}{3} \end{array}\right] \begin{matrix} +III \\ -III \\ \\ \end{matrix} \leftrightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & \frac{1}{3} \\ 0 & 1 & 0 & 1 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 & -1 & 0 & \frac{1}{3} \end{array}\right]$$

So the inverse is $\begin{bmatrix} 0 & -1 & \frac{1}{3} \\ 1 & 1 & -\frac{1}{3} \\ -1 & 0 & \frac{1}{3} \end{bmatrix}$.

2. (2 pts each) True or False? Briefly explain your answers.

(a) The only non-invertible 2×2 matrix is the matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

FALSE. Anything with rank less than 2 will be non-invertible. Some examples: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$, $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$.

(b) If A is invertible, then $A\vec{x} = \vec{b}$ always has a unique solution.

TRUE. This is essentially the definition of invertibility. A has to be a square ($n \times n$ matrix of rank n, so it has no rows of zeroes or free columns and must have a unique solution. This unique solution is $\vec{x} = A^{-1}\vec{b}$.

(c) If $A\vec{x} = \vec{b}$ is inconsistent, then A has more rows than columns.

FALSE. Any time rref(A) has a row of zeroes, there will be some $\vec{b}$ making the system inconsistent, so we could do this with a square matrix or even one with more columns. For example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

is inconsistent.

The appropriate statement would be "If $A\vec{x} = \vec{b}$ is inconsistent, then the rank of A is less than the number of rows of A."