QUIZ SOLUTIONS #5, 9/11/07

MATH 54, FALL 2007

1. (4 pts) Find the matrix for the tranformation given by dilation (i.e. expansion) by a factor of 5 followed by a rotation counterclockwise by $\frac{\pi}{2} = 90°$. (Hint: if you get stuck, figure out what happens to the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and what happens to the vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and then use this to find the matrix.)

The map is linear, so we only need to check it on $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.
$T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$ and $T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$. Thus the matrix is $\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$.

2. Setup: Draw some axes and draw the letter 'R' with its lower left corner at the origin (i.e. $(0, 0)$) and its left side along the y-axis.
(a) (2 pts) Now consider the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$. Draw what happens to the 'R' if you apply this transformation and briefly describe it in words.
The R is flipped over the x-axis and stretched vertically by a factor of two (but not horizontally).
(b) (2 pts) Find A^{-1} (the inverse matrix).

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right] \times \tfrac{1}{2} \leftrightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} \end{array}\right]$$

Thus the inverse matrix is $A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$
(c) (2 pts) Draw what would happen to the original 'R' from the setup (use a different location on the page for this drawing) if you apply the transformation A^{-1}.
The R is flipped over the x-axis and shrunk vertically by a factor of two (i.e. half the height), but not horizontally.