QUIZ SOLUTIONS #3, 9/4/07

MATH 54, FALL 2007

Show your work and justify your answers! Feel free to use both sides.

Name:

ID:

1. (4 pts) Find

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 6 & 0 \\ 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 6 & 0 \\ 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \cdot 0 + (-1) \cdot 1 + 3 \cdot 3 \\ 1 \cdot 0 + 6 \cdot 1 + 0 \cdot 3 \\ 0 \cdot 0 + 2 \cdot 1 + (-2) \cdot 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ -4 \end{bmatrix}$$

2. (a) (4 pts) What is the rank of the matrix

$$A = \begin{bmatrix} 0 & -1 & -5 \\ 1 & 4 & 0 \\ 0 & 3 & -2 \end{bmatrix} ?$$

We put the matrix into reduced row-echelon form:

$$\begin{bmatrix} 0 & -1 & -5 \\ 1 & 4 & 0 \\ 0 & 3 & -2 \end{bmatrix} \begin{matrix} \textit{swap} \\ \textit{swap} \\ \end{matrix} \leftrightarrow \begin{bmatrix} 1 & 4 & 0 \\ 0 & -1 & -5 \\ 0 & 3 & -2 \end{bmatrix} \times -1 \leftrightarrow$$

$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 5 \\ 0 & 3 & -2 \end{bmatrix} \begin{matrix} -4 \cdot II \\ \\ -3 \cdot II \end{matrix} \leftrightarrow \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & 5 \\ 0 & 0 & -17 \end{bmatrix} \times \tfrac{-1}{17} \leftrightarrow$$

$$\begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} +20 \cdot III \\ -5 \cdot III \\ \end{matrix} \leftrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

It has three leading ones, so its rank is three.

(b) (2 pts) Using the matrix A from part (a), if we choose a vector $\overrightarrow{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in $\mathbb{R}^3$ and try to solve $A\overrightarrow{x} = \overrightarrow{b}$ (where $\overrightarrow{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$), how many solutions will there be? To get full credit, briefly justify your answer (in words is fine).

The rank of the 3×3 matrix A is three, and so there will always be a unique solution. To see this. if we use an augmented matrix $[A|b]$ and put it into reduced row-echelon form, the left side will be as in part (a) and so, no matter what the right side ends up as, we'll get a unique solution ($x_1 =$ the first entry in the right-hand column of rref$(A|b)$, etc).