QUIZ SOLUTIONS #29, 12/6/07

MATH 54, FALL 2007

1. Find the Fourier sine series for $f(x) = 1$ on the interval $[0, \pi]$.

We have $1 = \sum_{n=1}^{\infty} a_n \sin(nx)$ where

$$\begin{aligned} a_n &= \frac{2}{\pi} \int_0^{\pi} 1 \cdot \sin(nx)\ dx \\ &= \frac{2}{\pi} \left[-\frac{1}{n} \cos(nx) \right]_0^{\pi} \\ &= \frac{2}{n\pi} \left(-(-1)^n - (-1) \right) \end{aligned}$$

Thus $a_n = \frac{4}{n\pi}$ for n odd and $a_n = 0$ for n even, so $1 = \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin((2k+1)x)$.

2. Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with the boundary conditions $u(0,t) = 0, u(\pi,t) = 0$ and $u(x,0) = 1$. (Your answer will involve an infinite sum.)

We have

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} a_n \sin(nx) e^{-n^2 t} \\ &= \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin((2k+1)x) e^{-(2k+1)^2 t} \end{aligned}$$