

QUIZ #27, 11/29/07

MATH 54, FALL 2007

Show your work and justify your answers! Feel free to use both sides.

Name:

1. (4pts) Determine all solutions to the boundary value problem $y'' - 4y = 0$ with $y(0) = 0$ and $y(1) = 1$.

The auxiliary equation is $r^2 - 4 = 0$, so $r = \pm 2$ and the general solution to $y'' - 4y = 0$ is $C_1e^{2t} + C_2e^{-2t}$.

We require that $0 = y(0) = C_1 + C_2$. Thus $C_2 = -C_1$.

We also require that $1 = y(1) = C_1e^2 + C_2e^{-2}$.

Thus $1 = C_1(e^2 - e^{-2})$, so $C_1 = \frac{1}{e^2 - e^{-2}}$ and $C_2 = -\frac{1}{e^2 - e^{-2}}$.

Thus $y(t) = \frac{1}{e^2 - e^{-2}}e^{2t} - \frac{1}{e^2 - e^{-2}}e^{-2t} = \frac{e^{2t} - e^{-2t}}{e^2 - e^{-2}}$.

2. (6pts) (a) For which values of $a > 0$ does the boundary value problem $y'' + a^2y = 0$, $y(0) = 0$, $y(\pi) = 0$ have a solution other than $y(t) = 0$?

The auxiliary equation is $r^2 + a^2 = 0$, so $r = \pm ai$. Thus the general solution is $C_1 \cos(at) + C_2 \sin(at)$.

We require that $0 = y(0) = C_1$, so $C_1 = 0$.

We also require that $0 = y(\pi) = C_2 \sin(a\pi)$.

If we have $C_2 = 0$, then we have only the trivial solution because we already know $C_1 = 0$. We know that $\sin(x) = 0$ precisely when $x = n\pi$ for n an integer, so we must have $a\pi = n\pi$; that is, a is an integer.

- (b) For which values of $b > 0$ does the boundary value problem $y'' - b^2y = 0$, $y(0) = 0$, $y(\pi) = 0$ have a solution other than $y(t) = 0$?

The auxiliary equation is $r^2 - b^2 = 0$, so $r = \pm b$. Thus the general solution is $C_1e^{bt} + C_2e^{-bt}$.

We require that $0 = y(0) = C_1 + C_2$.

We also require that $0 = y(\pi) = C_1e^{b\pi} + C_2e^{-b\pi}$.

We're looking for an element of the kernel of $\begin{bmatrix} 1 & 1 \\ e^{b\pi} & e^{-b\pi} \end{bmatrix}$. The determinant of this matrix is $e^{-b\pi} - e^{b\pi}$, which is zero only when $e^{-b\pi} = e^{b\pi}$. This happens only when $b = 0$ (because $e^x = e^y$ implies $x = y$), but we've assumed $b > 0$. Thus the matrix is invertible and thus we have only trivial solutions.