QUIZ SOLUTIONS #23, 11/13/07

MATH 54, FALL 2007

Show your work and justify your answers! Feel free to use both sides.

Name:

1. (6 pts) Rewrite $4y'''(t) + 3t^2y''(t) - \sin(t)y(t) = 2\sqrt{t}$ as a first-order system in the normal form $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$.

We let $x_1(t) = y(t)$, $x_2(t) = y'(t)$, and $x_3(t) = y''(t)$. Then we have $x_1'(t) = x_2(t)$, $x_2'(t) = x_3(t)$, and

$$x_3'(t) = \frac{\sin(t)}{4}x_1(t) - \frac{3t^2}{4}x_3(t) + \frac{\sqrt{t}}{2}.$$

Thus we have

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{\sin(t)}{4} & 0 & -\frac{3t^2}{4} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\sqrt{t}}{2} \end{bmatrix}$$

2. (4 pts) Find the general solution to $y''' + 9y' = 0$.

We try e^{rt}. This gives the auxiliary equation of $r^3 + 9r = 0$. This factors as $r(r^2 + 9)$, which has roots at 0, $3i$, and $-3i$. Thus the general solution is

$$y(t) = C_1 + C_2\cos(3t) + C_3\sin(3t)$$

(using $e^{0t} = 1$).