QUIZ #22, 11/8/07

MATH 54, FALL 2007

Show your work and justify your answers! Feel free to use both sides.

Name:

1. (5 pts) Consider the differential equation $y'' + y = x^2$. Note that $x^2 - 2$ is a solution.

(a) Find the general solution.

We first solve the complementary (i.e. homogeneous) equation $y'' + y = 0$. Trying e^{rx}, we get the auxiliary equation of $r^2 + 1 = 0$. This implies $r = \pm i$, so the complementary solutions are $C_1 \cos(x) + C_2 \sin(x)$.
The general solution is the particular solution plus the general complementary solution, so we get $y(x) = x^2 - 2 + C_1 \cos(x) + C_2 \sin(x)$.

(b) Is the set of solutions a subspace of the linear space C^∞ (i.e. the space of infinitely differentiable functions)?

It doesn't contain zero (due to the particular solution — i.e. due to the nonhomogeneous term), so the answer is no. In constrast, the set of solutions to a homogeneous linear differential equation do form a subspace; in particular, the complementary solutions above form a subspace, but when we "translate" them by adding on the particular solution, they no longer go through zero and are thus not a subspace.

2. (3 pts) Find the general solution to $y'' + 6y' + 10y = 0$.

We try e^{rx} and thus get the auxiliary equation $r^2 + 6r + 10 = 0$. Using the quadratic equation we get $r = -3 \pm i$. Thus the general solution is $C_1 e^{-3x} \cos(x) + C_2 e^{-3x} \sin(x)$.

3. (2 pts) True/False: $y'' - 3yy' + y = 0$ is a linear differential equation. (That is, $L[y] = y'' - 3yy' + y$ is a linear transformation from C^∞ to C^∞.) You needn't justify your answer.

FALSE. The yy' term is a product of two y-terms, making the equation nonlinear. For example, choosing $y(x) = x$, we have $L[cy] \neq cL[y]$, for $cL[x] = c(-3x + x) = -2cx$ while $L[cx] = -3c^2 x + cx$. For $c \neq 0, 1$ these are not equal.