# QUIZ #21, 11/6/07

MATH 54, FALL 2007

1. (7 pts) (a) Find all eigenvalues of $\begin{bmatrix} 1 & 1 \\ -10 & -1 \end{bmatrix}$. (You may express your answer in polar form or in coordinates.)

The characteristic polynomial is $\lambda^2 + 9$. Thus its roots $\pm 3i$ are the eigenvalues.

(b) Choose one of your eigenvalues from (a) and find its associated eigenvector(s).

Let's do $3i$. We have that $\mathrm{Eig}_{3i}$ is the kernel of $\begin{bmatrix} 1-3i & 1 \\ -10 & -1-3i \end{bmatrix}$. Let's row reduce this:

$$\begin{bmatrix} 1-3i & 1 \\ -10 & -1-3i \end{bmatrix} \times \tfrac{1}{1-3i} \leftrightarrow \begin{bmatrix} 1 & \frac{1+3i}{10} \\ -10 & -1-3i \end{bmatrix} +10 \cdot I \leftrightarrow \begin{bmatrix} 1 & \frac{1+3i}{10} \\ 0 & 0 \end{bmatrix}$$

(Using $\frac{1}{1-3i} = \frac{1}{1-3i}\frac{1+3i}{1+3i} = \frac{1+3i}{10}$ in the first step.)
The kernel of this is

$$\operatorname{span}\left(\begin{bmatrix} \frac{-1-3i}{10} \\ 1 \end{bmatrix}\right)$$

2. (3 pts) Find the general solution to $y'' + 3y' + 2y = 0$.

This is a second order linear equation, so we seek a two-dimensional family of solutions. We guess $e^{rx}$. Plugging this in gives

$$0 = r^2 e^{rx} + 3re^{rx} + 2e^{rx} = e^{rx}\left(r^2 + 3r + 2\right).$$

Because $e^{rx}$ is nonzero for some $x$ (in fact for all $x$), we must have

$$r^2 + 3r + 2 = 0$$

(the "auxiliary equation"). We can factor this as $(r+1)(r+2) = 0$ (or use the quadratic formula) to see that the roots are $r = -1$ and $r = -2$. Thus the solutions are $y_1 = e^{-x}$ and $y_2 = e^{-2x}$, so the general solution is

$$y(x) = C_1 e^{-x} + C_2 e^{-2x}.$$