QUIZ SOLUTIONS #2, 8/30/07

MATH 54, FALL 2007

Show your work and justify your answers! Feel free to use both sides.

Name:

ID:

1. (5 pts) Suppose we have a system of equations in 5 variables x_1, x_2, x_3, x_4, x_5 and we've taken it and rewritten it as an augmented matrix and we've gotten it into reduced row-echelon form as:

$$\left[\begin{array}{ccccc|c} 1 & 0 & 3 & 2 & 0 & 0 \\ 0 & 1 & 5 & 5 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Find all solutions. (In the matrix, we've used the expected order $x_1, x_2, \ldots, x_5$ for our variables.)

The variables corresponding to leading ones (i.e. x_1, x_2, and x_5) are our dependent variables and the others (x_2 and x_3) are our independent variables.
If we let $x_2 = s$ and $x_3 = t$, then $x_1 = -3s - 2t$, $x_2 = -2 - 5s - 5t$, and $x_5 = 4$.
Thus the solution set is

$$\{(-3s - 2t, -2 - 5s - 5t, s, t, 4) : s \text{ and } t \text{ are any real numbers}\}.$$

2. (5 pts) Solve the following system of equations by rewriting it as an augmented matrix and getting it into reduced row-echelon form.

$$\left\{\begin{array}{ccccccc} 2x_1 & + & 2x_2 & + & 4x_3 & = & -2 \\ x_1 & + & 2x_2 & + & x_3 & = & 0 \\ 2x_1 & - & 6x_2 & - & 4x_3 & = & -2 \end{array}\right\}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & -2 \\ 1 & 2 & 1 & 0 \\ 2 & -6 & -4 & -2 \end{array}\right] \begin{array}{c} \times\frac{1}{2} \\ \\ \\ \end{array} \leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & 2 & 1 & 0 \\ 2 & -6 & -4 & -2 \end{array}\right] \begin{array}{c} \\ -I \\ -2 \cdot I \end{array} \leftrightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & -8 & -8 & 0 \end{array}\right] \begin{array}{c} -II \\ \\ +8 \cdot II \end{array} \leftrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & -2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -16 & 8 \end{array}\right] \begin{array}{c} \\ \\ \times\frac{1}{8} \end{array} \leftrightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right] \begin{array}{c} -3 \cdot III \\ +III \\ \\ \end{array} \leftrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right]$$

Thus $x_1 = -\frac{1}{2}$, $x_2 = \frac{1}{2}$, and $x_3 = -\frac{1}{2}$.