QUIZ SOLUTIONS #17, 10/23/07

MATH 54, FALL 2007

Show your work and justify your answers! Feel free to use both sides.

Name:

1. (4 pts) What is the area of the parallelogram with corners at $(0,0)$, $(-3,2)$, $(4,1)$, and $(1,3)$?

This is $|\det(A)|$ for $A = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}$. This is $|-3 \cdot 1 - 4 \cdot 2| = |-11| = 11$.

2. (4 pts) What values can the determinant of an orthogonal transformation take? (Hint: Remember that an orthogonal tranformation A is one for which $A^T A = I_n$.) **Justify** your answer.

We have

$$1 = \det(I_n) = \det(A^T A) = \det(A^T)\det(A) = \det(A)\det(A) = \det(A)^2.$$

Thus $\det(A) = \pm 1$.
If we want to be completely rigorous, we should verify that both values are achieved. We can do this even with 1×1 matrices: $[1]$ and $[-1]$.

Another method is to say

$$|\det(A)| = ||v_1||\,||v_2^\perp|| \cdots ||v_n^\perp||$$

and that for an orthogonal matrix, the columns are already orthonormal, so we just get (with columns u_j)

$$|\det(A)| = ||u_1|| \cdots ||u_n|| = 1$$

so $\det(A) = \pm 1$ (and then argue as above that both values are achieved).

3. (2 pts) True/False (you needn't justify your answer): For every matrix A and scalar k, we have $\det(kA) = k \det(A)$.

FALSE. If A is an $n \times n$ matrix, then $\det(kA) = k^n \det(A)$ because it's linear in each row, and we have n rows to factor a k out of.
We could also do a quick check with the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and notice that its determinant is 4, not 2, while the determinant of I_2 is 1.