QUIZ SOLUTIONS #14, 10/11/07

MATH 54, FALL 2007

1. (4 pts) Find the matrix for orthogonal projection to the subspace of $\mathbb{R}^2$ spanned by $\begin{bmatrix} 4 \\ -3 \end{bmatrix}$.

We first get an orthonormal basis for the subspace. All we have to do is normalize: $\mathbf{u} = \begin{bmatrix} 4/5 \\ -3/5 \end{bmatrix}$. Thus we get that our matrix is:

$$\mathbf{u}\,\mathbf{u}^T = \begin{bmatrix} 4/5 \\ -3/5 \end{bmatrix} \begin{bmatrix} 4/5 & -3/5 \end{bmatrix} = \begin{bmatrix} 16/25 & -12/25 \\ -12/25 & 9/25 \end{bmatrix}.$$

2. True or False? Briefly justify your answers.

(a) (3 pts) If A is a skew-symmetric matrix (i.e. $A^T = -A$), then $(Av) \cdot w = v \cdot (Aw)$ for any pair of vectors v and w.

FALSE. $(Av) \cdot w = v \cdot (A^T w) = v \cdot (-Aw) = -v \cdot (Aw)$, so if $(Av) \cdot w$ is ever non-zero, they can't be equal. It should be clear that this is true, but I'll give an example just to make sure (using a skew-symmetric matrix):

$$\left(\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1$$

If you like, do out $v \cdot (Aw)$ in this case and double-check that you get -1.

(b) (3 pts) $A_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal (for any specific value of θ).

TRUE. Each column is a unit vector (because $\cos^2\theta + \sin^2\theta = 1$) and the columns are orthogonal.