QUIZ SOLUTIONS #12, 10/4/07

MATH 54, FALL 2007

1. (a) (3 pts) Project the vector $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ onto the line spanned by $\mathbf{v} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$.

$$\text{proj}_{\mathbf{v}}\mathbf{w} = \frac{\mathbf{w} \cdot \mathbf{v}}{||\mathbf{v}||^2}\mathbf{v} = \frac{6}{8}\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 3/2 \\ 0 \end{bmatrix}.$$

(b) (3 pts) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (you may leave a $\cos^{-1}$ in your answer). Is the angle acute, right, or obtuse?

The angle is

$$\cos^{-1}\left(\frac{\mathbf{w} \cdot \mathbf{v}}{||\mathbf{w}||\ ||\mathbf{v}||}\right) = \cos^{-1}\left(\frac{6}{\sqrt{6}\sqrt{8}}\right) = \cos^{-1}\left(\frac{6}{4\sqrt{3}}\right) = \cos^{-1}\left(\frac{6\sqrt{3}}{12}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}.$$

This is acute because $\mathbf{v} \cdot \mathbf{w}$ is positive.

2. (4 pts) Find a vector orthogonal to $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix}$. (Hint: If you get stuck, remember that you know how to solve systems of linear equations.)

We need a vector whose dot product with each of the above is zero. That's equivalent to solving the system of linear equations given by

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ -2 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & -3 & 0 \end{array}\right] \begin{array}{c} \\ +2I \\ -I \end{array} \leftrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 3 & 3 & 2 & 0 \\ 0 & 1 & -1 & -4 & 0 \end{array}\right] \begin{array}{c} \\ swap \\ swap \end{array} \leftrightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -4 & 0 \\ 0 & 3 & 3 & 2 & 0 \end{array}\right] \begin{array}{c} -II \\ \\ -3II \end{array} \leftrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 2 & 5 & 0 \\ 0 & 1 & -1 & -4 & 0 \\ 0 & 0 & 6 & 14 & 0 \end{array}\right] \begin{array}{c} \\ \\ \times\frac{1}{6} \end{array} \leftrightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 5 & 0 \\ 0 & 1 & -1 & -4 & 0 \\ 0 & 0 & 1 & 7/3 & 0 \end{array}\right] \begin{array}{c} +2III \\ +III \\ \\ \end{array} \leftrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & -5/3 & 0 \\ 0 & 0 & 1 & 7/3 & 0 \end{array}\right]$$

Thus, letting $x_4 = t$, we get that $\begin{bmatrix} -t/3 \\ 5t/3 \\ -7t/3 \\ t \end{bmatrix}$ works for any t. To give a specific non-zero vector, we plug in $t = 3$ (so we don't have fractions): $\begin{bmatrix} -1 \\ 5 \\ -7 \\ 3 \end{bmatrix}$.