

QUALIFYING EXAM SYLLABUS
MAY 4, 2009
ADAM BOOCHER

Committee: David Eisenbud, Bernd Sturmfels, Donald Sarason (Chair),
Katherine O'Brien O'Keeffe (outside member - English)

1. MAJOR TOPIC: COMMUTATIVE ALGEBRA

- Rings and modules: Noetherian rings/modules, graded rings/modules, localization, tensor products, Nakayama's lemma, Hilbert's Nullstellensatz
- Primary decomposition: prime avoidance, associated primes,
- Integral Extensions: going-up and going-down theorems, Noether normalization,
- Dimension Theory: Krull dimension, Hilbert polynomials, systems of parameters, principal ideal theorem,
- Monomial Orderings: Gröbner bases, Buchberger's algorithm, computations
- Resolutions and Homological Methods: the Koszul complex, syzygies, using resolutions to compute Hilbert Polynomial, Hilbert's syzygy theorem, depth, projective dimension, Auslander-Buchsbaum formula, Cohen-Macaulay rings.

References: Eisenbud, *Commutative Algebra with a View Toward Algebraic Geometry* Chapters 1-5, 8-13, 15, 17-18

2. MAJOR TOPIC: ALGEBRAIC GEOMETRY

- Schemes: affine, reduced, irreducible, regular, Noetherian, quasi-compact.
- Properties of morphisms: finite, finite type, closed and open immersions, separated, proper, projective, affine, quasi-compact.
- Sheaves of modules: quasi-coherent, coherent, locally free, invertible, ample, very ample, $\mathcal{O}(n)$.
- Divisors: Weil divisors, Cartier divisors, Picard group.
- Differentials
- Cohomology: Grothendieck vanishing theorem (statement). Serre's criterion for affineness. Čech cohomology. Cohomology of projective space, Serre Duality (statement)
- Curves: Riemann-Roch Theorem, Hurwitz's Theorem, embeddings in projective space, canonical embedding, elliptic curves (j -invariant and group law), Clifford's theorem.

References: Hartshorne, *Algebraic Geometry* Sections II.1-8, III.1-5, IV. 1-5.

3. MINOR TOPIC: COMPLEX ANALYSIS

- Holomorphic and meromorphic functions: Taylor and Laurent series, linear fractional transformations, Liouville's Theorem, Rouché's theorem.
- Complex Integration: Cauchy's theorem, Cauchy's integral formula, Morera's theorem, residue theorem.
- Riemann Mapping Theorem (statement)

References: Ahlfors, *Complex Analysis*, Chapters 1-4, Chapter 5, §1-3

4. OUTSIDE SUBJECT: OLD ENGLISH

- Translation of basic Old English